Selected Chapters from Philosophy of Science and Logic

Dutch Book Arguments

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LoPSE – http://lopsegdansk.blogspot.com/p/teaching.html

Introduction

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    3.2 Dutch Book arguments
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4. Today, we will look at Dutch Book arguments.

Dutch Book Arguments

1. **the main idea behind the argument**: Suppose that an agent's credences violate particular constraints (e.g. probabilism). We can then construct a Dutch Book against her. It is a set of bets, each of which the agent views as fair, but which together guarantee that she will lose money come what may.

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   1.1 We talk about bets and money losses, which are pragmatic consequences, and thus concern **pragmatic rationality**. We need a link to theoretic (epistemic) rationality.
   1.2 We talk about a collection of bets **each** of which the agent considers **fair**. The agent is myopic.

Fair Prices and Bets

A Nice Grandma Scenario

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   1.1 **Expected value:** $\frac{1}{100} \times (100 - 2) + \frac{99}{100} \times (-2) = -1$
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4. The fair price for the ticket that you pay is 1 zł i.e. $L = 1$. You lose 1 zł if the ticket does not win.
5. Notice that you are ready to sell a game (ticket) for a fair price or a higher price. You are also ready to buy a ticket for a fair price or a lower price

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3. We see that we can write the fair price L as $P(T_g) \times (W + L)$.
4. If we take situation in which the agent wins 1 zł ($W + L = 1$) if the proposition on which he bets is true and 0 zł otherwise, then the fair price will be $P(T_g) = L$ in some units of money.

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2. The value $\frac{L}{W+L}$ is a betting quotient and, for a fair bet, your degree of belief equals to the betting quotient.
3. A persons betting rate and their degree of confidence are connected. We can quantify someones personal probability using betting scenarios.

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Lottery Tickets

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Probability Axioms and Dutch Books

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   2.1 We will pro the Dutch Book Theorem case by case.

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   1.3 The bookie decides whether she wants to buy my ticket or sell me another ticket from the lottery for the fair price (I do not know if she buys or sells when I quote my price).

Non-Negativity

Non-Negativity: For any proposition X in $\mathcal{L}$, $P(X) \geq 0$.

1. Suppose that my $q(T_g) < 0$, where q is my betting quotient. A negative betting quotient means that I give away my ticket and pay to the person who takes it.

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T_g	my net payoff
true (wins)	$-1 + q(T_g)$
false (loses)	$q(T_g)$

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Finite Additivity: For any mutually exclusive propositions X and Y in $\mathcal{L}$, $P(X \vee Y) = P(X) + P(Y)$

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2. Suppose that $q(T_1 \vee T_2) < q(T_1) + q(T_2)$
3. The bookie will offer the agent the bet that pays 1 zł if T_1 and 0 zł otherwise for $q(T_1)$ and the bet that pays 1 zł if T_2 is true and 0 zł otherwise for $q(T_2)$.

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3. The bookie will offer the agent the bet that pays 1 zł if T_1 and 0 zł otherwise for $q(T_1)$ and the bet that pays 1 zł if T_2 is true and 0 zł otherwise for $q(T_2)$.
4. The bookie then buys the bet that will pay him 1 zł, if $(T_1 \vee T_2)$ is true and 0 zł otherwise, for the price of $q(T_1 \vee T_2)$.

Finite Additivity

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T_1	T_2	my net payoff
true (wins)	false (loses)	$1 - q(T_1) - q(T_2) + q(T_1 \vee T_2) - 1$
false (loses)	true (wins)	$-q(T_1) + 1 - q(T_2) + q(T_1 \vee T_2) - 1$
false (loses)	false (loses)	$-q(T_1) - q(T_2) + q(T_1 \vee T_2)$

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2. **Theorem** If an agent's credences violate at least one of the probability axioms, a Dutch Book can be constructed against her.
3. **Conclusion** Rational agents' credences do not violate the probability axioms.

Converse Dutch Book Argument

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   3.1 For instance, if an agent satisfies Non-Negativity, there will not be any propositions to which she assigns a negative credence, so we will not be able to construct a Book against her.

Some Criticism of Dutch Book Arguments

Credences and Betting prices

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2. Suppose that I am probabilistic, but my friend assigns to each proposition I believe a square root of my credence (he is not then probabilistic).
3. However, his betting prices are squares of his credences, so he cannot be Dutch Booked. Yet, he is not probabilistic.

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2. **Package Principle:** A rational agent's value for a package of bets equals the sum of her values for the individual bets it contains.

Thank you!