Conditional Credences and Conditionalisation

LoPSE seminars

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Introduction

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Ratio Formula: For any propositions P and Q in language $\mathcal{L}$, if $c(Q) > 0$ then

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Example - A. Hajek

Imagine throwing an infinitely fine dart at the [0,1] interval. Suppose that the probability measure for where the dart lands is uniform over the interval – the so-called 'Lebesgue measure'. What is the probability that the dart lands on the point $1/4$, given that it lands on either $1/4$ or $3/4$? $1/2$, surely. But the probability that the point lands on $1/4$ or $3/4$ is 0 according to the uniform measure.

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- For any mutually exclusive propositions P and Q in $\mathcal{L}$, $c(P \vee Q|R) = c(P|R) + c(Q|R)$.

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Notice: Conditional credence distribution must satisfy all the consequences of the probability axioms such as the **Negation rule**.

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\begin{aligned}
c(X) &= \sum_{i=1}^{n} c(X \& Q_i) && (2)\\
&= \sum_{i=1}^{n} c(Q_1) \times c(X|Q_i) && (3)\\
&= c(Q_1) \times c(X|Q_1) + \cdots + c(Q_n) \times c(X|Q_n). && (4)
\end{aligned}
$$

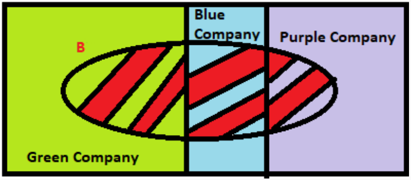
Law of Total Probability Example

Example 2

Suppose Adam Red wants to buy a hoverboard. Suppose that there exist only three manufacturers of that gadget. He buys a hoverboard from his local retailer, but, at home, he realises that there is no indication who manufactured the board.

He only knows that the market shares of the three companies (grren, blue, and purple) are 60%, 30%, and 10% respectively. Also, the probabilities that the battery will last less than an hour are 10%, 20%, and 60% respectively. What is the probability of the proposition B that the battery in Adam's hoverboard will last less than an hour?

Law of Total Probability Example – Solution



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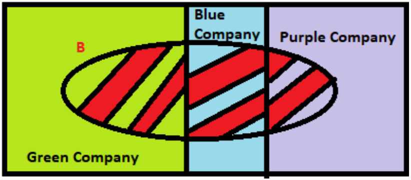

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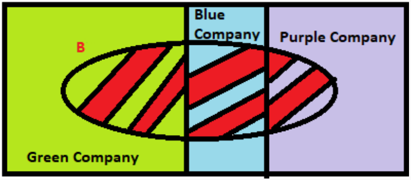

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Example 3

- $c(Q_1) = 0.6,\ c(Q_2) = 0.3, c(Q_3) = 0.1$
- $c(B|Q_1) = 0.1,\ c(B|Q_2) = 0.2, c(B|Q_3) = 0.6$
- $c(B) = 0.6 \times 0.1 + 0.3 \times 0.2 + 0.1 \times 0.6 = 0.18$

Dutch Book for the Ratio Formula

Example 3

Suppose that Raj has the following probability distribution (credence) c in two proposition P and Q:

P	Q	c
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1. We can read from the table that $c(P|Q) = \frac{c(P\&Q)}{c(Q)} = \frac{1/4}{2/4} = \frac{1}{2}$.
2. Suppose that Raj does not follow the ratio formula and assigns $c(P|Q) = 0.6$.

Dutch Book

1. The first ticket introduces a conditional bet. Conditional bets are priced using conditional credences. We sell this ticket to Raj.

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   If Q is true, this ticket entitles the bearer to 1$ if P is true and nothing otherwise. If Q is false, this ticket may be returned to the seller for a full refund of its purchase price.

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3. Raj's $c(\neg Q) = \frac{1}{2}$. We buy from Raj (pay Raj) for how much?

   Ticket 3

   If $\neg Q$ is true, this ticket entitles the bearer to 0.60$ and nothing otherwise.

Dutch Book – Net Gains

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How Raj will do when we consider all the bets together (what is his net gain in $ overall from all the bets)?

	$P\&Q$	$\neg P\&Q$	$\neg Q$
Ticket 1	0.4	-0.6	0
Ticket 2	-0.75	0.25	0.25
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Total Net Gain	-0.05	-0.05	-0.05

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Notice: No matter what happens, Raj's net gain in $ is negative, so he always loses money if he accepts all three bets.

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3. **Converse Dutch Book Theorem We Can Get:** As long as an agent's credences satisfy the probability axioms, she can't be Dutch Booked with the kind of Book we deployed against agents who violate the axioms.
4. **Notice:** Raj is probabilistic (his unconditional credences follow the probability axioms), but he is susceptible to a Dutch Book.

Conditionalisation

The Rule

Initially, you can see conditionalisation as an **updating rule** that says how to change your **current credences to new credences**, when you **learn** some evidence in the meantime. So it seems that conditionalisation connects credences at two moments (before learning and after learning) – it is sometimes called **diachronic rule**.

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Diachronic Conditionalisation: Let $\mathcal{B}$ be a set of propositions that includes possible evidence. Let c_t be your current credence on $\mathcal{B}$. Let $\mathcal{E} = \{E_1, \ldots, E_n\}$ be the partition of evidence that you can learn between t and some later moment $t+1$. Let c_{t+1} be your credence after learning $E_i \in \mathcal{E}$. The conditionalisation rule says that:

$$c_{t+1}(-) = c_t(-|E_i) = \frac{c_t(-\&E_i)}{c_t(E_i)}, \tag{5}$$

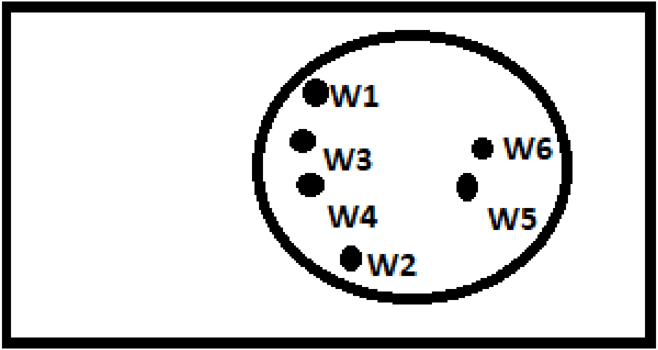
where $c_{t+1}(-)$ is of course defined only for $c_t(E_1) > 0$.

Conditionalisation – Example

Example 4

Suppose that Fatima and Ayesha play a game. Fatima rolls a die and gives Ayesha information about the outcome. She then updates her credences by conditionalisation until she gets the right answer. Suppose that the coin is fair and Ayesha has credence $\frac{1}{6}$ in the propositions that the outcome is 1, that the outcome is 2, etc.

Conditionalisation – Example



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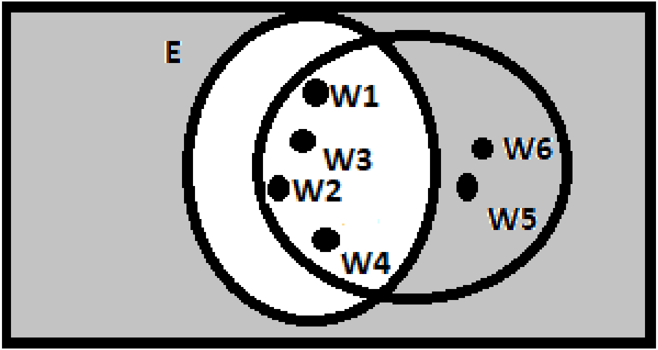

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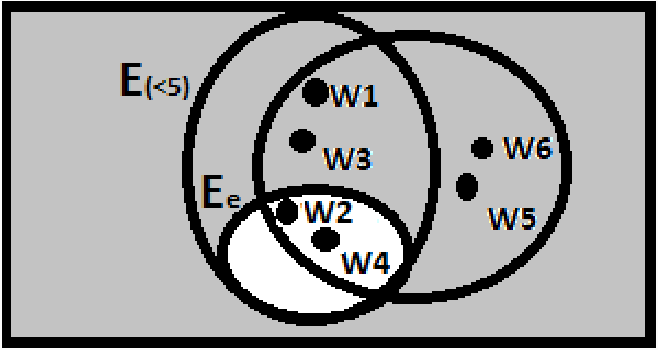

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 0.2.1 We have $\frac{\frac{1}{2}}{\frac{1}{2}} = 1$. Ayesha has credence 1 in the proposition that the die landed on 2.

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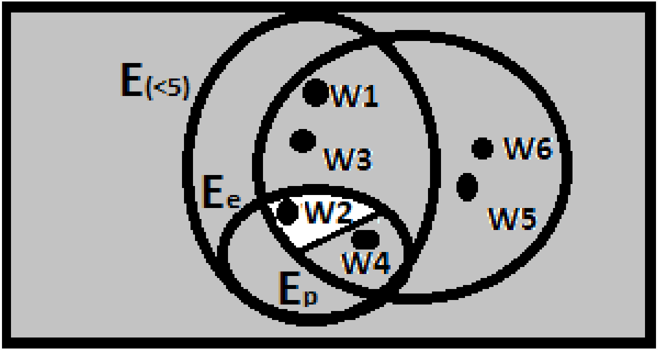

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   3.1 $c(X|T) = \frac{c(X\&T)}{c(T)} = \frac{c(X)}{1} = c(X)$

Conditionalisation – Dutch Book Strategy

Example 4

Suppose that Raj again has the following probability distribution (credence) c in two proposition P and Q:

P	Q	c_t
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1. He is probabilistic and satisfy the ration formula at his current state, so $c_t(P|Q) = \frac{1}{2}$.
2. But when he learns Q, he will assign new credence to P which will be, say, $c_{t+1} = 0.6$ – Raj does not follow conditionalisation.

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   If $P\&Q$ is true, this ticket entitles the bearer to 1\$ and nothing otherwise.

3. Raj's $c(\neg Q) = \frac{1}{2}$, so he sells for that price.

   Ticket 3

   If $\neg Q$ is true, this ticket entitles the bearer to 0.60\$ and nothing otherwise.

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   1.3 How much will Raj pay for that ticket at $t+1$?

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How Raj will do when we consider all the bets together (what is his net gain in $ overall from all the bets)?

	$P\&Q$	$\neg P\&Q$	$\neg Q$
Ticket if Q learned	0.4	-0.6	0
Ticket 1	-0.75	0.25	0.25
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Notice: No matter what happens, Raj's net gain in $ is negative, so he always loses money if he accepts all three bets.

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3. In Dutch Book strategies, the bookie needs to know what the updated agent's credence will be to either sell or buy the initial two bets (Ticket 1 and Ticket 2).
4. To keep the idea that the agent and the bookie know the same things, the agent needs to know that she will violate conditionalisation. How is that possible?

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2. If Dutch Strategies reveal any kind of rational tension, it seems to be one that exists not between an agent's t and $t+1$ credences, but instead between her t credences and her t plans for updating going forward.

Planned Conditionalisation

We can formulate conditionalisation not as a diachronic updating rule but a rational plan for updating credences upon receiving evidence.

Planned Conditionalisation: Let $\mathcal{B}$ be a set of propositions that includes possible evidence. Let c_t be your current credence on $\mathcal{B}$. Let $\mathcal{E} = \{E_1, \ldots, E_n\}$ be the partition of evidence that you can learn between t and some later moment $t+1$. Let R be a plan how to update upon receiving evidence from $E \in \mathcal{E}$. If you were to receive evidence E_i and if $c(E_i) > 0$, then R would exhort you to adopt credence function

$$c_{t+1}(-) = c_t(-|E_i) = \frac{c_t(-\&E_i)}{c_t(E_i)}. \tag{6}$$

Conditionalisation and Reflection

Original Reflection Principle

Let c_c be your current credence function, c_t your credence function at some future time t, and X a proposition. Then, if you are rational, for any H and t:

$$c_c(X|c_t(X) = x) = x. \tag{7}$$

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2. **Notice**: Reflection is a synchronic norm – credences are assigned at the same (current) time; they just happen to be credences about some future moment.

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Indicator function:

$$I_X(w) = \begin{cases} 1 & w \in X \\ 0 & w \notin X \end{cases}$$

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1. You can always write credence in a proposition X as expected value of X's indicator function – X is treated as a random variable:

Indicator function:

$$I_X(w) = \begin{cases} 1 & w \in X \\ 0 & w \notin X \end{cases}$$

Credence as Expected Value of I_X:
$c(X) = \mathbb{E}(I_X) = c(X) \times 1 + c(\neg X) \times 0$

Generalized Reflection

Pettigrew: For any random variable X and future t, the expected value of X relative to you current credence function c_c must lie in the span of the foreseeable expected values of X relative to $c_{R(E,\mathcal{E})}$.

$$c_c(X) = \sum_{E \in \mathcal{E}} c_c(E) c_{R(E,\mathcal{E})}(X) \tag{8}$$

Thank you!