Conditional Credences and Conditionalisation

Philosophy of Science

Pavel Janda
pavel.janda.early@gmail.com

LoPSE – http://lopsegdansk.blogspot.com
https://www.paveljandaphil.com/

Introduction

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Ratio Formula: For any propositions P and Q in language $\mathcal{L}$, if $c(Q) > 0$ then

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Example - A. Hajek

Imagine throwing an infinitely fine dart at the [0,1] interval. Suppose that the probability measure for where the dart lands is uniform over the interval – the so-called 'Lebesgue measure'. What is the probability that the dart lands on the point $1/4$, given that it lands on either $1/4$ or $3/4$? $1/2$, surely. But the probability that the point lands on $1/4$ or $3/4$ is 0 according to the uniform measure.

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- For any mutually exclusive propositions P and Q in $\mathcal{L}$, $c(P \vee Q|R) = c(P|R) + c(Q|R)$.

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Notice: Conditional credence distribution must satisfy all the consequences of the probability axioms such as the **Negation rule**.

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\begin{aligned}
c(X) &= \sum_{i=1}^{n} c(X \& Q_i) && (2)\\
&= \sum_{i=1}^{n} c(Q_1) \times c(X|Q_i) && (3)\\
&= c(Q_1) \times c(X|Q_1) + \cdots + c(Q_n) \times c(X|Q_n). && (4)
\end{aligned}
$$

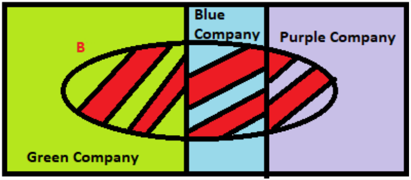
Law of Total Probability Example

Example 2

Suppose Adam Red wants to buy a hoverboard. Suppose that there exist only three manufacturers of that gadget. He buys a hoverboard from his local retailer, but, at home, he realises that there is no indication who manufactured the board.

He only knows that the market shares of the three companies (grren, blue, and purple) are 60%, 30%, and 10% respectively. Also, the probabilities that the battery will last less than an hour are 10%, 20%, and 60% respectively. What is the probability of the proposition B that the battery in Adam's hoverboard will last less than an hour?

Law of Total Probability Example – Solution



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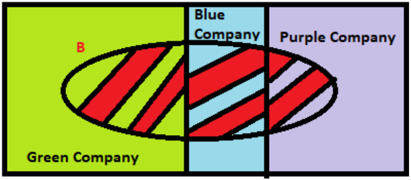

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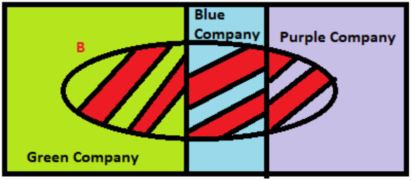

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Example 3

- $c(Q_1) = 0.6,\ c(Q_2) = 0.3, c(Q_3) = 0.1$
- $c(B|Q_1) = 0.1,\ c(B|Q_2) = 0.2, c(B|Q_3) = 0.6$
- $c(B) = 0.6 \times 0.1 + 0.3 \times 0.2 + 0.1 \times 0.6 = 0.18$

Dutch Book for the Ratio Formula

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Suppose that Raj has the following probability distribution (credence) c in two proposition P and Q:

P	Q	c
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1. We can read from the table that $c(P|Q) = \frac{c(P\&Q)}{c(Q)} = \frac{1/4}{2/4} = \frac{1}{2}$.
2. Suppose that Raj does not follow the ratio formula and assigns $c(P|Q) = 0.6$.

Dutch Book

1. The first ticket introduces a conditional bet. Conditional bets are priced using conditional credences. We sell this ticket to Raj.

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   If Q is true, this ticket entitles the bearer to 1$ if P is true and nothing otherwise. If Q is false, this ticket may be returned to the seller for a full refund of its purchase price.

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3. Raj's $c(\neg Q) = \frac{1}{2}$. We buy from Raj (pay Raj) for how much?

   Ticket 3

   If $\neg Q$ is true, this ticket entitles the bearer to 0.60\$ and nothing otherwise.

Dutch Book – Net Gains

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How Raj will do when we consider all the bets together (what is his net gain in $ overall from all the bets)?

	$P\&Q$	$\neg P\&Q$	$\neg Q$
Ticket 1	0.4	-0.6	0
Ticket 2	-0.75	0.25	0.25
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Total Net Gain	-0.05	-0.05	-0.05

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Notice: No matter what happens, Raj's net gain in $ is negative, so he always loses money if he accepts all three bets.

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3. **Converse Dutch Book Theorem We Can Get:** As long as an agent's credences satisfy the probability axioms, she can't be Dutch Booked with the kind of Book we deployed against agents who violate the axioms.
4. **Notice:** Raj is probabilistic (his unconditional credences follow the probability axioms), but he is susceptible to a Dutch Book.

Conditionalisation

The Rule

Initially, you can see conditionalisation as an **updating rule** that says how to change your **current credences to new credences**, when you **learn** some evidence in the meantime. So it seems that conditionalisation connects credences at two moments (before learning and after learning) – it is sometimes called **diachronic rule**.

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Diachronic Conditionalisation: Let $\mathcal{B}$ be a set of propositions that includes possible evidence. Let c_t be your current credence on $\mathcal{B}$. Let $\mathcal{E} = \{E_1, \ldots, E_n\}$ be the partition of evidence that you can learn between t and some later moment $t+1$. Let c_{t+1} be your credence after learning $E_i \in \mathcal{E}$. The conditionalisation rule says that:

$$c_{t+1}(-) = c_t(-|E_i) = \frac{c_t(-\&E_i)}{c_t(E_i)}, \tag{5}$$

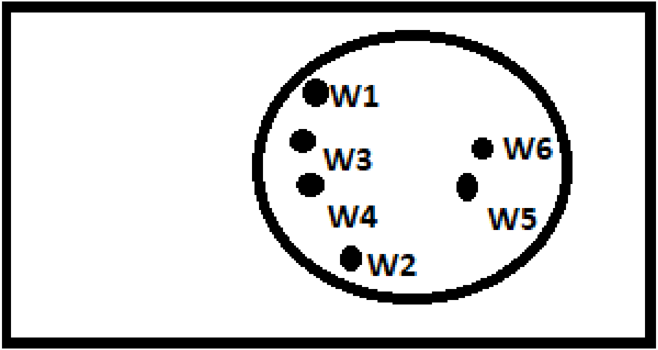
where $c_{t+1}(-)$ is of course defined only for $c_t(E_1) > 0$.

Conditionalisation – Example

Example 4

Suppose that Fatima and Ayesha play a game. Fatima rolls a die and gives Ayesha information about the outcome. She then updates her credences by conditionalisation until she gets the right answer. Suppose that the die is fair and Ayesha has credence $\frac{1}{6}$ in the propositions that the outcome is 1, that the outcome is 2, etc.

Conditionalisation – Example



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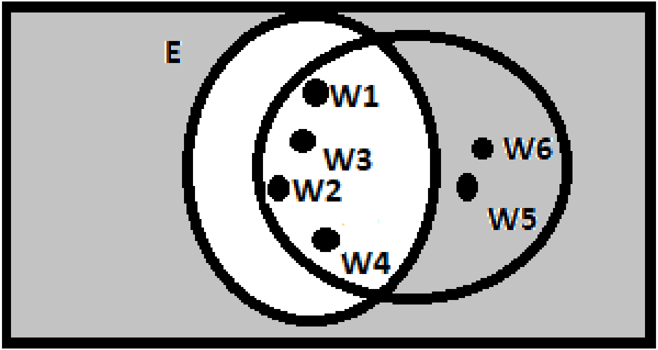

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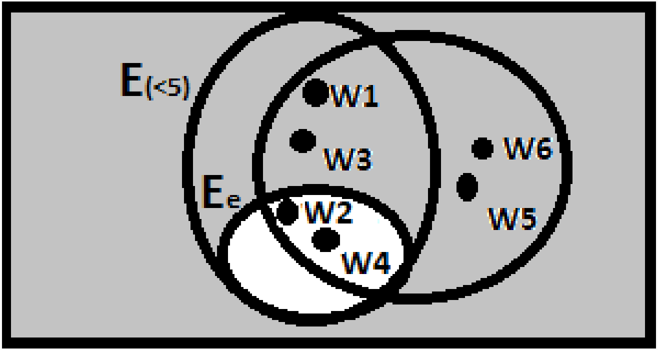

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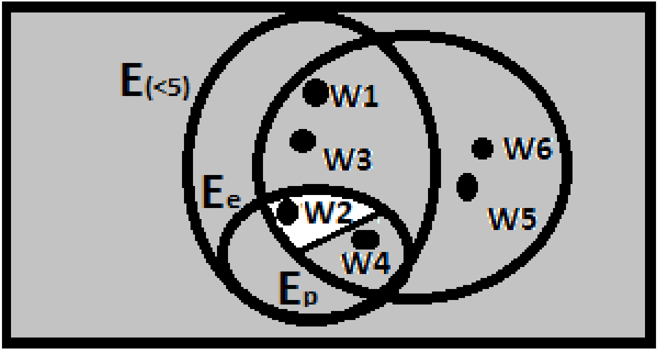
Conditionalisation – Example



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Consequences of Conditionalisation

Diachronic Conditionalisation: Let $\mathcal{B}$ be a set of propositions that includes possible evidence. Let c_t be your current credence on $\mathcal{B}$. Let $\mathcal{E} = \{E_1, \ldots, E_n\}$ be the partition of evidence that you can learn between t and some later moment $t+1$. Let c_{t+1} be your credence after learning $E_i \in \mathcal{E}$. The conditionalisation rule says that:

$$c_{t+1}(-) = c_t(-|E_i) = \frac{c_t(-\&E_i)}{c_t(E_i)}, \tag{6}$$

where $c_{t+1}(-)$ is of course defined only for $c_t(E_1) > 0$.

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1. When you learn new evidence, one consequence of conditionalisation is that you learn it with certainty.

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Proof for credence 1 in X

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2. **Different perspective**: Once you rule out some possible worlds by learning, they remain ruled out forever as long as you conditionalise.

Forgetting Your Evidence

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2. information that is time or agent dependent
   2.1 I am sure that the proposition that today is Thursday is true. But in 24 hours, I will not be sure any more that the proposition that today is Thursday is true.

Memory Loss - Talbott's Spaghetti dinner

If we assume that I had spaghetti for dinner one year ago (on March 15, 1989), then where t_1 is 6:30pm on March 15, 1989; t_2 is 6:30pm on March 15, 1990; and S is the proposition that Talbott had spaghetti for dinner on March 15, 1989...if I am assumed to satisfy Temporal Conditionalization at every time from t_1 to t_2, it must be the case that I be as certain of S at t_2 as I was at t_1 ... But, in fact (and I knew this at t_1), if at t_2 I am asked how probable it is that I had spaghetti for dinner on March 15, 1989, the best I can do is to calculate a probability based on the relative frequency of spaghetti dinners in my diet one year ago.

Talbott, W.J. Two Principles of Bayesian Epistemology, *Philosophical Studies*, 62, 1991, p. 135–150.

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3. But there is another example show that we need more than just an assumption that ideally rational agents do not forget.

Memory Loss - Two Paths to Shangri La

There are two paths to Shangri La, the Path by the Mountains, and the Path by the Sea. A fair coin will be tossed by the guardians to determine which path you will take: if heads you go by the Mountains, if tails you go by the Sea. If you go by the Mountains, nothing strange will happen: while traveling you will see the glorious Mountains, and even after you enter Shangri La, you will forever maintain your memories of that Magnificent Journey. If you go by the Sea, you will revel in the Beauty of the Misty Ocean. But, just as you enter Shangri La, your memory of this Beauteous Journey will be erased and be replaced by a memory of the Journey by the Mountains.

Arntzenius, F. Some Problems for Conditionalization and Reflection, *The Journal of Philosophy*, 100(7), 2003, p. 356–370.

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    2.3 But is it irrational to assign credence bigger than 0 to the contingent possibility that one forgot something at some point in the past?

Slippery Evidence

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7. Possible worlds, as we have discussed them so far, are too coarse-grained descriptions to give us answer.

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7. Possible worlds, as we have discussed them so far, are too coarse-grained descriptions to give us answer.
8. We need to introduce more fine-grained entities than classical possible worlds.

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5. We can call classical possible worlds **uncentred worlds** and propositions represented by sets of uncentred possible worlds as **uncentred porpositions**.

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6. Yet centred propositions present the agent with moving targets; as she gains (or loses) information, their truth-values may change as well.

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        1.3.1 Take a particular uncentered world (call it u). Consider all the centres associated with that world you entertained as live possibilities at t_i. Assign credence 0 to any centred world incompatible with E. Then take the credence assigned to u in Step 1 above, and distribute it among the remaining centred worlds associated with that uncentred world.

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2. **Notice:** The second step is underspecified. It does not say how we should distribute credences among the centred worlds.

How Do We Update on Centred Propositions?

Example - Titlebaum p. 380

At t_1, you have no idea what time it is. You are wearing a watch that you are 60% confident is running reliably. The rest of your credence is divided equally between the possibility that the watch is entirely stopped, and the possibility that it is running but is off by an exactly an hour (perhaps due to daylight savings). You then glance at your watch, notice that it is running (the second hand is moving), and that it currently reads 1pm.

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Example Comtinued - Titlebaum p. 380

stopped	initial c	running and shows 1pm – step 1
stopped	0.2	0
reliable	0.6	0.75
1 hour off	0.2	0.25

How Do We Update on Centred Propositions?

Example Comtinued - Titlebaum p. 380

stopped	step 1
stopped	0
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Example Comtinued - Titlebaum p. 380

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Example Comtinued - Titlebaum p. 380

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Example Comtinued - Titlebaum p. 380

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Example Comtinued - Titlebaum p. 380

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Example Comtinued - Titlebaum p. 380

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   2.2 $c(w_2, 12pm) = 0$, $c(w_2, 1pm) = 0.75$, $c(w_1, 2pm) = 0$.
   2.3 $c(w_2, 12pm) = 0 - 0.25$, $c(w_2, 1pm) = 0$, $c(w_1, 2pm) = 0 - 0.25$

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3. **Notice:** Regularity is not entailed by the probability axioms.

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   3.1 Can we defend Conditionalisation somehow?
   3.2 We can, for example, use foundationalist epistemologies – e.g. sense data and phenomenal experiences are certain.

Jeffrey Conditionalisation

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4. What do we do then?

Jeffrey Conditionalisation

Jeffrey Conditionalisation Given any t_i and t_j with $i < j$, any X in $\mathcal{L}$, and a finite partition $B_1, B_2, \ldots, B_n$ in $\mathcal{L}$ whose elements each have non-zero cr_i,

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Suppose that you learn B_1 with certainty from the finite partition $B_1, B_2, \ldots, B_n$.

$$\begin{aligned} cr_j(X) &= cr_i(X|B_1) \times 1 + \cdots + cr_i(X|B_n) \times 0 \\ &= cr_i(X|B_1) \end{aligned} \quad (8)$$

Classical Example

Example

The agent inspects a piece of cloth by candlelight, and gets the impression that it is green, although he concedes that it might be blue or even (but very improbably) violet. If G, B, and V are the propositions that the cloth is green, blue, and violet, respectively, then the outcome of the observation might be that, whereas originally his degrees of belief in G, B, and V were 0.3, 0.3, and 0.4, his degrees of belief in those same propositions after the observation are 0.7, 0.25, and 0.5.

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The worry is that even if we grant the existence of a sense datum for each potential learning experience, the quality of that sense datum might not be representable in a proposition to which the agent could assign certainty

Concrete Example – Titlebaum p. 160

Example Continued

Suppose that:

$c_1(G) = 0.3 \qquad c_1(B) = 0.3 \qquad c_1(V) = 0.4$

and

$c_1(mine|G) = 0.8 \qquad c_1(mine|B) = 0.5 \qquad c_1(mine|V) = 0$

and

$c_2(G) = 0.7 \qquad c_2(B) = 0.25 \qquad c_2(V) = 0.05$

so by Jeffrey conditionalisation

$$\begin{aligned} c_2(mine) &= c_1(mine|G)c_2(G) + c_1(mine|B)c_2(B) + c_1(mine|V)c_2(V) \\ &= 0.80 * 0.70 + 0.50 * 0.25 + 0 * 0.05 \\ &= 0.685 \end{aligned} \tag{9}$$

Concrete Example – Titlebaum p. 160

Example Continued

$$c_2(mine) = c_1(mine|G)c_2(G) + c_1(mine|B)c_2(B) + c_1(A|V)c_2(V) \tag{10}$$

Use the Law of Total probability on $cr_2(mine)$

$$c_2(mine) = c_2(mine|G)c_2(G) + c_2(mine|B)c_2(B) + cr_2(A|V)c_2(V) \tag{11}$$

It must be the case that

$$c_1(mine|G) = c_2(mine|G), c_1(mine|B) = c_2(mine|B) \dots \tag{12}$$

Rigidity

Rigidity: For any A in $\mathcal{L}$ and any B_m in $B_1, B_2, \ldots, B_n$

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Jeffrey Conditionalisation using a particular partition $B_1, B_2, \ldots, B_n$ is appropriate only when the agent's credences conditional on the B_m remain constant across two times.

Issues with Jeffrey Conditionalisation

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2. Jeffrey Conditionalisation is **not commutative**. It matters in which order we learn evidence (classical conditionalisation is commutative).

Caveat: Bayes' Theorem

1. Remember conditional credences? Suppose that we have some hypothesis H and evidence E, the $c(H|E)$ is the probabilistic credence that H is true given that we suppose that E is true.

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Bayes' Theorem: $c(H|E) = \frac{c(E|H)*c(H)}{c(E)}$ for $c(E) \neq 0$.

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Non-Commutativity of Jeffrey Conditionalisation

Raven Example

Suppose I see a bird at twilight, and I clearly identify it to be a raven. Because of the difficulty in identifying the bird's color in the gloom, I do not observe that e – "The bird is black" – but the effect of my experience is to raise my confidence in e from 0.75 to 0.99. If h is "All ravens are black" and my background beliefs include that the bird is a raven.

M. Lange, Is Jeffrey Conditionalization Defective by Virtue of Being Non-Commutative? Remarks on the Sameness of Sensory Experiences, *Synthese*, 123(3), 2000, pp. 394

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$c_{new}(h) = c_{old}(h|e) * c_{new}(e) + c_{old}(h|\neg e) * c_{new}(\neg e)$.

0.2 We know that $c_{new}(e) = 0.99$ and $c_{new}(\neg e) = 0.01$.

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Suppose I see a bird at twilight, and I clearly identify it to be a raven. Because of the difficulty in identifying the bird's color in the gloom, I do not observe that e – "The bird is black" – but the effect of my experience is to raise my confidence in e from 0.75 to 0.99. If h is "All ravens are black" and my background beliefs include that the bird is a raven.

M. Lange, Is Jeffrey Conditionalization Defective by Virtue of Being Non-Commutative? Remarks on the Sameness of Sensory Experiences, *Synthese*, 123(3), 2000, pp. 394

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2. **Notice:** In this example, the result of successive applications of Jeffrey conditionalization firstly to $c(e) = 0.99$ and then $c(e) = 0.8$ is indeed the same as the result of applying Jeffrey conditionalization once first to $c(e) = 0.8$, as if the 0.99 experience had never occurred (this does not happen always).

Slippery Evidence – Problem for Jeffrey Conditionalisation

Raven Example – modified Titlebaum p. 378

0.1 I look out of a window. I see drops of water on my window. Experience directly influences my credences over the partition containing the centred propositions 1.) it's raining now and 2.) it's not raining now (maybe my neighbour is just watering his flowers.) I want to set my unconditional credence in the proposition S that it rains on (a specific) Sunday.

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0.2 Suppose that $e(\neg e)$ stands for 'it's (not) raining now'. By Jeffrey conditionalisation:

$$c_{new}(S) = c_{old}(S|e) * c_{new}(e) + c_{old}(S|\neg e) * c_{new}(\neg e) \tag{14}$$

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0.3 Whatever $c_{new}(e)$ and $c_{new}(\neg e)$ are, by the Bayes' Theorem:

$$c_{old}(S|e) = \frac{c_{old}(e|S) * c_{old}(S)}{c_{old}(e)} \tag{15}$$

The value of $c_{old}(S|e)$ has to remain the same (by rigidity).

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   1.1 Suppose 'now' means Sunday. Further, suppose that my initial credence in S (before the experience) is 0.5, the experience raised my credence in e from 0.6 to 0.8, and $1 > c_{old}(e|S) > 0$, then:

$$c_{old}(S|e) = \frac{c_{old}(e|S) * c_{old}(S)}{c_{old}(e)} = \frac{c_{old}(e|S) * 0.5}{0.6}, \qquad (16)$$

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   so $1 > c_{old}(S|e) > 0$.

   1.2 Suppose 'now' means Monday. On Monday you know whether it rained on that specific Sunday, so $c_{old}(S|e) = 1$ if it rained and $c_{old}(S|e) = 0$ if ir did not rain.

Thank you!